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Comparison of Classical, Generalized and Hybrid Integral Transforms: A Review

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ABSTRACT

Integral transforms are among the most powerful mathematical tools for solving differential, integral, and integro-differential equations by transforming them into simpler algebraic forms. This review presents a comprehensive overview of the evolution, mathematical foundations, operational properties applications, and recent developments of classical, generalized, and hybrid integral transforms. A comparative analysis is provided to highlight their similarities, differences, advantages, and limitations in terms of computational efficiency, convergence, and engineering applications. Recent advances in generalized transform theory, hybrid analytical techniques, fractional calculus, artificial intelligence-assisted computation, and unified transform frameworks are also discussed. Furthermore, the paper identifies current research gaps and outlines future research directions aimed at developing unified mathematical theories, efficient computational algorithms.

Introduction:

Integral transforms provide systematic techniques for converting differential, integral, and integro-differential equations into algebraic equations that are easier to analyze and solve. Since the eighteenth century, integral transforms have played a fundamental role in mathematics, engineering, physics, and computational sciences by simplifying complex mathematical models and enabling analytical solutions to a wide variety of practical problems [1], [11].

The earliest and most influential integral transforms are the Laplace and Fourier transforms, which remain the foundation of transform theory. The Laplace transform, developed from the work of Pierre-Simon Laplace, is widely used for solving ordinary differential equations, transient response analysis, electrical circuits, control systems, heat transfer, and vibration problems because it naturally incorporates initial conditions into the transformed equations [1], [15]. Similarly, the Fourier transform, introduced through the pioneering work of Joseph Fourier, has become indispensable in frequency-domain analysis, signal processing, image processing, communication systems, acoustics, quantum mechanics, and electromagnetic theory [2], [12].

Other classical transforms, including the Mellin, Hankel, Hilbert, and Laplace–Carson transforms, have also made significant contributions to solving specialized mathematical problems. These transforms possess well-established theoretical properties such as linearity, convolution theorems, inverse transformations, and convergence conditions, making them reliable analytical tools for solving boundary value problems and partial differential equations [11], [12].

Although classical transforms have achieved remarkable success, the increasing complexity of modern engineering and scientific problems has exposed certain limitations. Many contemporary mathematical models involve nonlinear, multidimensional, stochastic, fuzzy, and fractional differential equations that are difficult to solve efficiently using only classical transform methods [13]. These challenges motivated researchers to develop generalized integral transforms with modified kernel functions and improved operational properties.

A major milestone in this direction was the introduction of the Sumudu Transform by Watugala in 1993 [3]. Unlike the Laplace transform, the Sumudu transform preserves the physical dimensions of variables, making it particularly useful in engineering applications. Subsequently, several generalized transforms—including the Natural Transform [4], Elzaki Transform [5], Aboodh Transform [6], Kamal Transform [7], Mohand Transform [8], and Shehu Transform [9]—were proposed to simplify the solution of ordinary, partial, and fractional differential equations while improving computational efficiency.

During the last decade, research has shifted toward hybrid and unified transform frameworks. Instead of introducing isolated transforms with slightly modified kernels, researchers have focused on constructing generalized mathematical formulations capable of generating multiple transforms as special cases. One of the most significant developments is the BA Transform, introduced by Belafhal, El Aitouni, and Usman in 2024, which unifies several classical and generalized transforms within a single parameterized framework [10]. This approach has reduced redundancy in transform theory and provided a common mathematical basis for comparative studies.

Another important trend is the integration of integral transforms with fractional calculus, which has become an active area of research due to its ability to model memory-dependent and hereditary phenomena. Fractional differential equations have found applications in viscoelastic materials, porous media, biological systems, financial mathematics, and control engineering. Generalized transforms have been successfully extended to solve equations involving Caputo, Riemann–Liouville, and other fractional derivatives [13], [14].

In addition, hybrid analytical–numerical approaches such as the Generalized Integral Transform Technique (GITT) have significantly enhanced the applicability of transform methods to nonlinear engineering problems. By combining analytical transforms with numerical approximation techniques, GITT provides accurate and computationally efficient solutions for heat transfer, fluid mechanics, convection–diffusion equations, and biomedical transport phenomena [15].

Despite the rapid development of generalized and hybrid transforms, several challenges remain. Many recently proposed transforms possess similar operational properties and differ only in kernel parameterization, raising questions regarding mathematical novelty, computational advantages, and practical significance. Furthermore, systematic comparative studies, standardized benchmarking methods, and software implementations are still limited [10], [12]. These issues highlight the need for comprehensive review studies that critically evaluate the evolution, applications, strengths, and limitations of classical, generalized, and hybrid transform techniques.

2. Literature Review

Integral transform theory has evolved considerably over the last two centuries. The development of classical transforms established the foundation of operational calculus, while modern research has introduced generalized, hybrid, and unified transform frameworks to address increasingly complex mathematical models. This section reviews the evolution of integral transforms and highlights recent research trends.

2.1 Evolution of Classical Integral Transforms:

The origin of integral transforms dates back to the pioneering work of Pierre-Simon Laplace in the eighteenth century. The Laplace transform became one of the most important analytical tools for solving ordinary differential equations, electrical circuit problems, control systems, and transient phenomena because it converts differential equations into algebraic equations while naturally incorporating initial conditions [1], [11].

In the nineteenth century, Joseph Fourier introduced the Fourier transform, which revolutionized the analysis of heat conduction and periodic phenomena. Today, it is extensively used in signal processing, image processing, communication systems, quantum mechanics, acoustics, and electromagnetic theory [2], [12].

Other classical transforms, including the Mellin, Hankel, Hilbert, and Laplace–Carson transforms, were subsequently developed for solving specialized mathematical and engineering problems. These transforms possess rigorous mathematical properties such as linearity, inversion formulas, convolution theorems, and convergence conditions, making them indispensable tools in applied mathematics and engineering [11], [12].

By the end of the twentieth century, classical transforms had become well-established analytical techniques with extensive theoretical development, software implementation, and practical applications. However, their effectiveness is limited for many nonlinear, multidimensional, and fractional-order problems, motivating the development of generalized transform methods [13].

2.2 Development of Modern Integral Transforms:

The period 1993–2019 marked a major advancement in integral transform theory with the introduction of several generalized transforms designed to overcome the limitations of classical methods. A significant milestone was the Sumudu Transform, proposed by Watugala (1993), which preserves the physical dimensions of variables and has been successfully applied to differential equations, heat transfer, fluid mechanics, and control engineering [3].

To combine the advantages of the Laplace and Sumudu transforms, Khan and Khan (2008) introduced the Natural Transform, providing a unified framework for solving differential equations and boundary-value problems [4]. Later, the Elzaki Transform proposed by Elzaki (2011) simplified the solution of ordinary and partial differential equations and found applications in wave propagation, diffusion, and heat conduction [5].

Additional generalized transforms, including the Aboodh, Kamal, Mohand, and Shehu transforms, further expanded the family of Laplace-type transforms by modifying kernel functions and operational properties [6]–[9]. These transforms proved effective for solving nonlinear, fractional, and engineering problems while offering computational simplicity.

The rapid growth of fractional calculus during this period also contributed significantly to transform research.

Generalized transforms were extended to solve equations involving Caputo and Riemann–Liouville fractional derivatives, leading to applications in viscoelasticity, diffusion, biological systems, finance, and control engineering [13], [14].

Overall, the years 1993–2019 transformed integral transform theory from a collection of classical methods into a rapidly expanding field of generalized analytical techniques.

2.3 Recent Hybrid and Generalized Integral Transforms:

The period 2020–2026 represents a new phase in integral transform research, focusing on generalized, hybrid, and unified transform frameworks rather than introducing isolated transforms. These approaches improve the solution of nonlinear, fractional, stochastic, and multidimensional differential equations [10].

One of the most significant developments is the BA Transform, proposed by Belafhal, El Aitouni, and Usman (2024), which unifies several transforms—including the Laplace, Sumudu, Natural, Elzaki, Aboodh, Mohand, Kamal, and Shehu transforms—within a single mathematical framework [10].

Hybrid transform techniques, such as Laplace–Sumudu, Laplace–Fourier, and the Generalized Integral Transform Technique (GITT), have also gained considerable attention. These methods combine analytical and numerical approaches to solve complex engineering problems involving heat transfer, fluid flow, and nonlinear partial differential equations [15].

Recent studies also explore the integration of artificial intelligence, symbolic computation, and machine learning with transform techniques to improve computational efficiency and automate transform selection for complex mathematical models.

2.4 Comparative Analysis of Existing Studies:

Most published studies propose new transforms or demonstrate their application to specific differential equations. Comparatively few studies systematically evaluate different transforms using common mathematical or computational criteria.

Comparative investigations indicate that many generalized transforms retain the operational properties of the Laplace transform, differing mainly in kernel functions or scaling parameters [10], [12]. Classical transforms remain the preferred choice for conventional engineering applications because of their mature mathematical theory and extensive computational support. In contrast, generalized and hybrid transforms provide greater flexibility for solving nonlinear, fractional, and multidimensional problems [13].

Recent unified frameworks demonstrate that many generalized transforms are mathematically related rather than entirely independent analytical tools, encouraging researchers to develop comprehensive transform theories instead of introducing new kernel functions with only minor modifications [10].

2.5 Research Trends:

Current research shows several important trends in integral transform theory:

- Development of generalized and unified transform frameworks.
- Increasing applications of fractional calculus.
- Growth of hybrid analytical–numerical methods.
- Integration with artificial intelligence and symbolic computation.
- Expansion into multidimensional and multiphysics problems.
- Development of efficient computational algorithms and software.

These trends demonstrate that integral transform theory continues to evolve in response to the growing complexity of modern scientific and engineering problems.

3. Mathematical Foundations and Operational Properties:

3.1 General Definition of Integral Transform

An integral transform converts a function from one domain into another by means of a kernel function. In general, an integral transform is expressed as

$$F(s) = \int_a^b K(s,t) f(t) dt$$

where, (K(s,t)) is the kernel function, (f(t)) is the original function, and (F(s)) is the transformed function [11], [12].

3.2 Mathematical Definitions of Transforms

Laplace Transform

Let f(t) be a function of the variable t which is defined for all positive values of t. Let s be the real constant then Laplace transform is defined as

$$F(s) = \int_0^\infty e^{-st} f(t) dt\text{, if it exists.}$$

Sumudu Transform

The Sumudu Transform with parameter p of f(x) denoted by S[f(x)]=F(p) and is given by

$$S\{f(x) = F(p) = -\frac{1}{p}\int_0^\infty e^{-\frac{x}{p}} f(x) dx$$

for parameter $p > 0$.

Fourier Transform

The Fourier transform with parameter s of f(t) is denoted by $F[f(t)] = F(s)$ and is given by

$$F[f(t)] = F(s) = \int_{-\infty}^{\infty} e^{-ist} f(t)\, dt \ ,$$

for parameter s.

Natural Transform

The natural transforms combine the Laplace and Sumudu transforms

$$N\{f(t)\} = \int_0^{\infty} e^{-ist} f(ut) dt$$

where s and u are transform parameters.

Elzaki transform

The Elzaki transform is defined as $E\{f(t)\} = v \int_0^{\infty} f(t) \mathrm{e}^{-t/v}\, dt$ where $v > 0$

Shehu transform

The Shehu transform generalizes both the Laplace and Sumudu transforms.

$$H\{f(t)\} = \int_0^{\infty} e^{-st/u} f(t)\, dt,$$

where $s > 0$, $u > 0$

Aboodh Transform

The Aboodh transform is defined by

$$A\{f(t)\} = \frac{1}{s}\int_0^{\infty} e^{-st} f(t)\, dt$$

Mohand Transform

The Mohand transform is given by

$$M\{f(t)\} = \mathrm{v}^2 \int_0^{\infty} e^{-vt} f(t) dt \ ,$$

where $v > 0$

Kamal Transform

The Kamal transform is defined as

$$K\{f(t)\} = \int_0^{\infty} e^{-t/v} f(t)\, dt \ ,$$

where $v > 0$

BA Transform

The recently proposed BA Transform provides a generalized mathematical framework capable of generating numerous existing integral transforms as special cases. The general form is

$$B\{f(t)\} = \int_0^{\infty} K_{\alpha,\beta}(s,t) f(t) dt \ ,$$

where $K_{\alpha,\beta}(s,t)$ is parameterized kernel.

Generalized Integral Transform

A generalized integral transform can be written as

$$G\{f(t)\} = \int_0^{\infty} K(s,t;\alpha,\beta,\gamma) f(t)\, dt$$

where $K(s,t;\alpha,\beta,\gamma)$ is kernel and α,β,γ are parameters

By selecting suitable parameter values, many classical and modern transforms become special cases. Generalized transforms provide greater mathematical flexibility, improved convergence properties, simpler operational rules, and enhanced applicability to nonlinear and fractional differential equations. The mathematical definitions illustrate the evolution of integral transforms from the classical Laplace and Fourier transforms to modern generalized and unified transform frameworks. Although these transforms differ in their kernel functions and operational formulations, they share the common objective of simplifying mathematical models through transformation into an alternative domain. The emergence of generalized and hybrid transforms demonstrates an ongoing effort to improve computational efficiency, analytical flexibility, and applicability to nonlinear, multidimensional, and fractional-order systems. These definitions provide the theoretical foundation for the detailed comparative analysis.

Despite differences in kernel functions, these transforms satisfy important operational properties such as linearity, differentiation, inverse transformation, and convolution, making them effective tools for solving mathematical models [3]–[10].

4. Comparative Analysis of Classical, Generalized, and Hybrid Integral Transforms

4.1: Table

Transform	Kernel Function	Domain	Convergence Requirement	Major Advantages	Limitations	Nature
Laplace	e^{-st}	$0 \le t < \infty$	Exponential order	Simple operational rules, initial conditions included	Restricted mainly to causal systems	Classical
Fourier	$e^{-i\omega t}$	$-\infty < t < \infty$	Absolutely integrable function	Excellent frequency analysis	Requires stronger convergence conditions	Classical
Sumudu	$-\frac{1}{p}e^{-\frac{x}{p}}$	$0 \le t < \infty$	Exponential growth condition	Preserves physical dimensions	Limited theoretical literature	Generalized
Natural	e^{-st}	$0 \le t < \infty$	Similar to Laplace	Combines Laplace and Sumudu properties	Less software support	Generalized
Elzaki	$ve^{-t/v}$	$0 \le t < \infty$	Mild exponential condition	Simplified derivative formulas	Applications still developing	Generalized
Shehu	$e^{-st/u}$	$0 \le t < \infty$	Parameter dependent	Suitable for fractional models	Limited numerical algorithms	Generalized
Aboodh	$\frac{e^{-st}}{s}$	$0 \le t < \infty$	Laplace-type convergence	Computational simplicity	Mathematical theory still expanding	Generalized
Mohand	$v^2\, e^{-vt}$	$0 \le t < \infty$	Exponential order	Efficient operational rules	Less engineering applications	Generalized
Kamal	$e^{-t/v}$	$0 \le t < \infty$	Exponential order	Easy inverse transformation	Limited comparative studies	Generalized
Mahgoub	ve^{-vt}	$0 \le t < \infty$	Exponential order	Simple mathematical formulation	Requires further theoretical validation	Generalized
Yang	$e^{-t/u}$	$0 \le t < \infty$	Exponential growth	Effective for fractional calculus	Limited computational software	Generalized
BA Transform	Parameterized exponential kernel	Depends on selected parameters	Depends on kernel parameters	Unified framework for multiple transforms	Recently introduced; requires further investigation	Hybrid/unified

4.2 Table

Property	Laplace	Fourier	Modern Generalized Transforms
Linearity	yes	yes	yes
Differentiation	yes	yes	yes
Integration	yes	yes	yes
Convolution	yes	yes	Most possess
Shifting Property	yes	yes	Most possess
Initial Value Theorem	yes	Limited	Many possess
Final Value Theorem	yes	No	Many possess
Fractional Calculus	Moderate	Limited	yes

4.3 Comparison of Major Transforms:

Classical, generalized, and hybrid integral transforms each possess distinct strengths and limitations depending on the type of mathematical problem.

Classical transforms, such as the Laplace and Fourier transforms, have an excellent mathematical foundation supported by rigorous theories, well-established operational properties, and extensive software implementations. They are highly effective for solving ordinary and partial differential equations and have broad applications in engineering and applied sciences. However, their performance is moderate when dealing with nonlinear, fractional, and multiphysics problems.

Generalized transforms, including the Sumudu, Natural, Elzaki, Aboodh, Mohand, Kamal, and Shehu transforms, also provide excellent performance for solving differential equations while offering excellent capability for nonlinear and fractional differential equations. They generally exhibit very good computational efficiency, although their mathematical foundation is still developing and software support remains limited.

Hybrid transforms combine the advantages of two or more existing transforms, resulting in excellent computational efficiency and greater flexibility. They perform particularly well in solving nonlinear, fractional, and multiphysics problems and are increasingly applied to multidisciplinary engineering models. However, like generalized transforms, they currently have limited software implementation and require further theoretical development.

Overall, classical transforms remain the benchmark because of their strong mathematical foundation and widespread computational support [11], [12]. Generalized transforms extend their applicability to nonlinear and fractional-order systems, whereas hybrid transforms offer the greatest flexibility and computational efficiency for complex multidisciplinary problems [10], [13].

Example:1 Laplace transform

Solve the ordinary differential equations

$$\frac{dy}{dx}+3y=6 \quad ,y\left(0\right)=2$$

Using the Laplace transform.

Solution: Taking Laplace transform gives

$$L\{\frac{dy}{dt}\}=sY(s)-y(0)$$

Gives $sY(s)-2+3Y(s)=s^6$

Therefore $(s+3)Y(s)=\frac{6}{s}+2$

Hence $Y(s)=\frac{6}{s(s+3)}+\frac{2}{s+3}$

$$Y(s)=s^2$$

Taking inverse Laplace transform $y(t)=2$

Example 2: Fourier Transform

Find the Fourier transform of $f(x)=e^{-a|x|}, \qquad a>0$

Solution: Using Fourier transform

$$F(\omega)=\int_{-\infty}^{\infty} e^{-a|x|}e^{-i\omega x}dx$$

$$\int_{0}^{\infty} e^{-ax}cos(\omega x)dx=\frac{a}{a^2+\omega^2}$$

Therefore $F(\omega)=\frac{2a}{a^2+\omega^2}$

Fourier Transform converts a time/space-domain function into its frequency spectrum and is fundamental in signal processing.

Example 3: Sumudu Transform

Solve: $y'+y=1, y(0)=0$

Solution: Taking Sumudu transform $S\{y'(t)\}=\frac{Y(u)-y(0)}{u}$

$$\frac{Y(u)}{u}+Y(u)=1$$

$$Y(u)=\frac{u}{1+u}$$

Taking inverse Sumudu transform

$$y(t)=1-e^{-t}$$

Unlike Laplace Transform, the Sumudu Transform preserves the physical units of the original function.

Example 4: Elzaki Transform

Solve: $y'+2y=0, \ y(0)=1$

Solution: Using Elzaki Transform

$$E\{y'(t)\}=\frac{T(v)}{v}-vy(0)$$

$$\frac{T(v)}{v}-v+2T(v)=0$$

$$T(v)=\frac{v^2}{1+2v}$$

The Elzaki Transform simplifies initial value problems while avoiding some scaling difficulties encountered in Laplace Transform.

Example 5: Hybrid Transform

Consider the one-dimensional heat equation

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, \quad u(x,0)=f(x)$$

Applying hybrid transform

$$sU(k,s)-F(k)=-k^2U(k,s)$$

$$U(k,s)=\frac{F(k)}{s+k^2}$$

Taking inverse transform

$u(x,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} f(k)e^{-k^2 t}e^{ikx}dk$ This is the general solution of heat equation.

Using the Fourier convolution theorem

$$u(x,t)=\frac{1}{4\pi t}\int_{-\infty}^{\infty} f(\xi)exp[\frac{(x-\xi)^2}{-4t}]d\xi$$

At t = 0 , $\lim_{t\to\infty}\frac{1}{\sqrt{4\pi t}}e^{\frac{-(x-\xi)^2}{4t}}=\delta(x-\xi)$

$$u(x,0)=\int_{-\infty}^{\infty} f(\xi)\delta(x-\xi)d\xi = f(x)$$

Which satisfies initial condition.

5. Applications of Classical, Generalized, and Hybrid Integral Transforms

Integral transforms have wide-ranging applications in mathematics, engineering, and the physical sciences by simplifying complex differential equations into algebraic forms.

Major application areas include:

- Engineering: control systems, structural analysis, vibration, robotics, and aerospace engineering [1], [15].
- Physics: heat conduction, wave propagation, quantum mechanics, electromagnetics, and diffusion problems [2], [11].
- Fluid Dynamics: Navier–Stokes equations, boundary layer flow, porous media, and magnetohydrodynamics [15].
- Heat and Mass Transfer: transient heat conduction, diffusion, thermal radiation, and heat exchangers [11].
- Electrical Engineering: circuit analysis, power systems, transmission lines, and filter design [1].
- Signal and Image Processing: spectrum analysis, communication systems, image reconstruction, speech processing, and medical imaging [2].
- Fractional Calculus: viscoelasticity, anomalous diffusion, biological systems, finance, and control engineering [13], [14].
- Biomedical Engineering: blood flow modeling, ECG/EEG analysis, MRI reconstruction, and drug delivery systems.
- Artificial Intelligence: intelligent signal processing, pattern recognition, symbolic computation, and automated transform selection.

Classical transforms remain the preferred choice for conventional engineering problems, whereas generalized and hybrid transforms are increasingly applied to nonlinear and fractional-order mathematical models.

6. Recent Developments

Research during 2020–2026 has focused on generalized, hybrid, and unified transform frameworks.

Major developments include:

- BA Transform (2024): A unified framework encompassing several existing transforms through a parameterized kernel function [10].
- General Integral Transform: Demonstrates that many modern transforms belong to a common family of Laplace-type transforms.
- Hybrid Integral Transforms: Combinations such as Laplace–Sumudu and Laplace–Fourier improve computational efficiency.
- Generalized Integral Transform Technique (GITT): Combines analytical and numerical methods for solving nonlinear engineering problems [15].
- Fractional Integral Transforms: Extended to Caputo and Riemann–Liouville derivatives for solving memory-dependent systems [13].
- Artificial Intelligence Integration: AI-assisted symbolic computation, transform selection, and numerical inversion algorithms are emerging research areas.

These developments indicate a transition from isolated transform methods toward unified computational frameworks.

7. Research Gaps

Despite considerable progress, several research challenges remain:

1] Lack of a unified mathematical theory for generalized transforms.

2] Limited comparative benchmarking and performance evaluation.

3] Insufficient software implementation and computational libraries.

4] Similarity among many generalized transforms.

5] Limited industrial and multidisciplinary applications.

Addressing these issues will improve the practical applicability and acceptance of generalized and hybrid integral transforms.

8. Future Scope

Future research should focus on:

1] Developing new hybrid and generalized transforms.

2] Solving nonlinear and fractional differential equations.

3] Extending multidimensional transform techniques.

4] Integrating transforms with artificial intelligence, machine learning, and symbolic computation.

5] Developing efficient numerical algorithms and software packages.

6] Expanding applications in biomedical engineering, renewable energy, IoT, finance, quantum computing, and climate modeling.

These directions will further strengthen the role of integral transforms in modern computational mathematics.

9. Conclusion

Integral transforms continue to be among the most powerful analytical tools for solving differential, integral, and integro-differential equations. Classical transforms, particularly the Laplace and Fourier transforms, remain fundamental because of their rigorous mathematical foundations and extensive applications in engineering, physics, and applied mathematics.

The development of generalized transforms, including the Sumudu, Natural, Elzaki, Aboodh, Mohand, Kamal, and Shehu transforms, has significantly expanded the scope of transform methods to nonlinear, multidimensional, and fractional differential equations. More recently, hybrid and unified frameworks, such as the BA Transform and GITT, have improved computational efficiency and demonstrated that many generalized transforms share common mathematical structures.

Comparative analysis shows that no single transform is universally superior. Instead, classical, generalized, and hybrid transforms should be regarded as complementary mathematical tools, with the choice depending on the nature of the governing equations and computational requirements. Future research should emphasize unified mathematical theories, standardized benchmarking, efficient software implementation, and interdisciplinary applications involving artificial intelligence, machine learning, and high-performance computing.

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